predictions for $\bar{\tau}_t$ and ϵ_m that are compatible in the outer wall region gives evidence of the internal consistency in the general surface renewal modeling approach.

As indicated in the analysis, the mean frequency $\bar{S}$ of fluid exchange between the turbulent core and the wall region has been assumed to be equal to the mean frequency $\bar{s}$ of the turbulent burst process; i.e., $U^{*2}/(\bar{S}\nu)=133$. This prediction for $\bar{S}$ (or $\bar{s}$) is consistent with much of the experimental data obtained by flow visualization and anemometry techniques for the mean periodicity in turbulent fluctuations in wall turbulence.

The surface rejuvenation model, which accounts for the effect of the unreplenished layer of fluid on the transport of momentum associated with the turbulent burst phenomenon, is shown in the present Note to provide a basis for the prediction of Reynolds stress $\bar{\tau}_t$ and eddy diffusivity ϵ_m within the wall region. The potential usefulness of this general approach is underscored by: 1) the basic consistency between the predictions for ϵ_m and experimental data within the wall region, 2) the appropriateness of the limiting predictions for $\bar{\tau}_t$ and ϵ_m as y^+ becomes large and small, 3) the consistency of the direct and alternative formulation approaches, and 4) compatibility of the predictions for $\bar{s}$ (or $\bar{S}$) with experimental measurements within the wall region. Furthermore, this general approach to modeling wall turbulence can be applied to heat and mass transfer and can be adapted to problems involving transitional turbulence, acceleration, variable properties, natural convection, and other factors.

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Empirical Estimates of Gust Loads on Finite Wings

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I. Introduction

ESTIMATES of the aerodynamic loads on finite aircraft wings in atmospheric gusts are a necessary part of the design exercise for modern aircraft. A variety of theoretical computations based on vortex and/or doublet lattice panel methods[1,2] are the most common means of arriving at these estimates, although experimental data from wind tunnel testing also contribute in the assessment of specific designs.

The availability of wind tunnel data[3,4] for unsteady lift and pitching moment on a variety of finite wing planforms in oscillatory gusts presented the opportunity of devising an empirical relationship which describes the variation of oscillating aerodynamic forces as a function of frequency, freestream velocity, wing sweep angle, and aspect ratio over a range of from 0 to 70 deg in sweep angle and from 1 to 8 in aspect ratio. This Note presents the empirical relationship and its derivation from the raw data.

A gust tunnel of novel design[5] has been used to measure the oscillating lift and pitching moments on a variety of wing planforms subjected to incident oscillatory vertical gusts of varying frequency, amplitude, and freestream velocity. Table 1 displays salient details of the eight wing planforms that were tested. They range from a rectangular wing (A) of aspect ratio 6 to wing F with aspect ratio 4 and a sweep angle of 45 deg. Two of the planforms (G and H) are delta wings with aspect ratios of 2 and 1, respectively. All the wings had straight trailing and leading edges and no center body at the wing root. The nondelta planforms were built with symmetric NACA 0010 air foil sections. The wing tips were shaped as surfaces of revolution by rotating the upper airfoil surface profile through 180 deg about the tip chord line. The delta wings were constructed as plane surfaces with sharp symmetric wedge shaped leading edges having an included angle of 30 deg. All the conventional planforms were untapered with the exception of wing D (aspect ratio 6, half chord line sweep of 19.7 deg). The empirical relationships detailed later exclude the effects of wing taper. However, wing D is still included because experimental data shows[3] that the taper has a relatively small effect on the gust forces as compared to an identical untapered wing.

The oscillatory gust force data are presented in Figs. 1 and 2 as ratios of $\Delta C_L/\bar{w}_g$ and $\Delta C_M/\bar{w}_g$ against a frequency parameter, $\bar{\omega}=\omega c/U$; where ΔC_L and ΔC_M are amplitudes of lift and pitching moment coefficients, $\bar{w}_g$ is the amplitude of gust angle or nondimensionalized "downwash" velocity, ω is the radian gust frequency, c is the wing geometric mean chord, and U is the mean freestream velocity. The results presented here are nondimensionalized with respect to the amplitude of gust oscillation because the original data indicated that for gust incidence variations below stall, the measured force amplitudes behaved linearly with increasing gust amplitude. Thus the results displayed in Figs. 1 and 2 are independent of the wing angle of incidence away from stall. In order to ensure this linearity, the boundary layer on the upper and lower surfaces of the nondelta wings was tripped by sandpaper roughness close to the leading edges. The two delta

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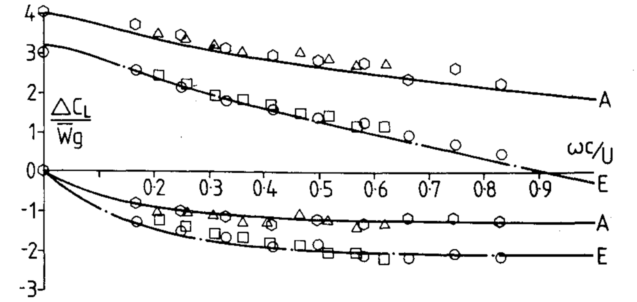

Fig. 1 Oscillating lift-force data and empirically fitted curves for wings A ($A=6$, $\lambda_{1/2}=0$ deg) and E ($A=6$, $\lambda_{1/2}=45$ deg).

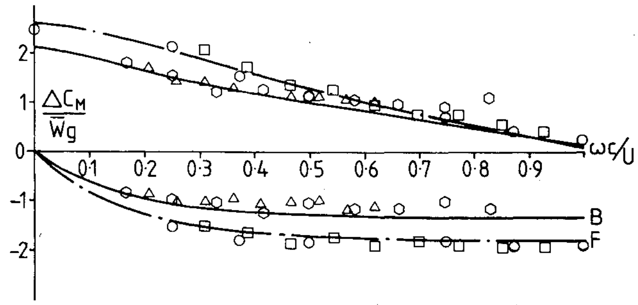

Fig. 2 Oscillating pitching-moment data and empirically fitted curves for wings B ($A=6$, $\lambda_{1/2}=22.5$ deg) and F ($A=4$, $\lambda_{1/2}=45$ deg).

wings had to be given special consideration due to the additional nonlinear vortex lift component obtained with these planforms. As a result, for both delta wings G and H, only the lift measurements at 0 deg angle of incidence are included. The delta wing pitching moment data are excluded from consideration here due to their unusual nature (see Ref. 4 for details).

Although the original data were measured with a gust approaching the test wings at a velocity of $0.61U$, the data presented here have been converted (see Ref. 3) to the more conventional situation of the gusts approaching the wings at the freestream velocity. Figures 1 and 2 present the raw data as plotted points with the results at two freestream velocities (12.43 and 20.00 m/s) differentiated by the circle or hexagon and square or triangle symbols, respectively. Both the lift and pitching moment data are given in terms of in-phase and out-of-phase components measured relative to the freestream gust oscillation at the wing-root quarter chord point. Reference 3 gives further information on the methods of measurement.

It is worth noting that the test data used here are based on measured wing forces in sinusoidal gusts. The results of further tests[6] indicate that the principle of superposition may be used to extrapolate the wing gust forces to a nonsinusoidal incident gust if each of its Fourier harmonic components are used as separate inputs to the empirical relationship given here. The resultant force response to each of the Fourier components can then be summed to yield the total force.

Steady flow measurements of lift and pitching moment curve slopes were used to deduce the "zero frequency" points in the culrves presented here. These steady flow data show good agreement with the results of inviscid theory (pressented in Ref. 7) based on linearized lifting surface computations. The data sheets of Ref. 7 are used to derive a zero frequency datum in the empirical gust response expressions presented here. These expressions then model the variation of gust induced forces with gust frequency, planform sweep angle, and aspect ratio from the zero frequency datum.

II. Data Manipulation

The plotting symbols in Figs. 1 and 2 indicate the kind of trends encountered in the raw data. The planform angle of sweep has a dominant effect on the phase lags embedded in the results whereas aspect ratio tends to influence the force

Table 1 Details of wing planforms

Wing designation	Aspect ratio	Sweep angle, $\lambda_{1/2}/dg$	Taper ratio, t	rms error in $\Delta C_L/\bar{w}_g$ for curve fit
A	6	0	1	0.15
B	6	22.5	1	0.15
C	8	22.5	1	0.18
D	6	19.7	0.5	0.13
E	6	45.0	1	0.15
F	4	45.0	1	0.19
G	2	45.0	0	0.14
H	1	63.4	0	0.07

amplitude magnitudes only with little effect on the phase angles. All the data are characterized by zero slope for the in-phase curves and a zero value for the out-of-phase curves at the zero frequency abscissa. Furthermore, at low frequencies ($\omega c/U<0.2$) the measured in-phase and out-of-phase force and moment amplitudes change rapidly with increasing frequency parameter before leveling out to a virtually constant rate of change with frequency parameter.

The choice of curve-fitting function was governed primarily by this change in the measured force characteristics across the frequency parameter range. The expressions that follow incorporate exponential functions and describe families of curves typified by the experimental data being considered.

$$\left.\frac{\Delta C_L}{\bar{w}_g}\right|_{\text{in phase}} \text{ or } \left.\frac{\Delta C_M}{\bar{w}_g}\right|_{\text{in phase}} = a+b\bar{\omega}+[(dh-b)\bar{\omega}+h]e^{-d\bar{\omega}} \tag{1}$$

$$\left.\frac{\Delta C_L}{\bar{w}_g}\right|_{\text{out of phase}} \text{ or } \left.\frac{\Delta C_M}{\bar{w}_g}\right|_{\text{out of phase}} = -g(1-e^{-f\bar{\omega}}) \tag{2}$$

where a, b, d, f, g, and h are constants selected during the curve-fitting procedure. The form of the expressions is such as to ensure zero slope for the in-phase lift and moment amplitudes and zero values for the out-of-phase force amplitudes at zero frequency parameter in all cases.

The curve-fitting procedure was implemented through a computer code which searches through and selects appropriate values of the parameters a, b, d, f, g, and h based on the criterion of minimizing the root mean square error between the empirical expressions and the data points. This procedure is repeated for all the planforms and the global variation of parameters b, d, f, and g with aspect ratio and sweep angle are further evaluated as algebraic expressions. The values of a and h, however, are predetermined by the steady flow lift and pitching moment curve slopes obtained from the data sheets of Ref. 7. Equations (1) and (2) are such that for $\bar{\omega}=0$, the numerical value $(a+h)$ equals the steady flow lift and pitching moment curve slopes which are equivalent to parameters $\Delta C_L/\bar{w}_g$ and $\Delta C_M/\bar{w}_g$ at $\omega=0$. The curves of Figs. 1 and 2, as well as Table 1, illustrate the rms error magnitudes encountered in the curve-fitting procedure. For the overall curve fitting, an average rms error of 0.145 is obtained for $\Delta C_L/\bar{w}_g$ and 0.22 for $\Delta C_m/\bar{w}_g$.

III. Gust Load Function

The variation of the in-phase lift coefficient amplitude per unit gust amplitude ($\Delta C_L/\bar{w}_g$) as a function of frequency parameter ($\bar{\omega}=\omega c/U$) is given by Eq. (1) with the constants taking the values $a=0.82C_{L\alpha}$, $b=-A\,(0.27+0.024\,A\tan\lambda_{1/2})$, $d=4+A^3/32$, and $h=0.13C_{L\alpha}$. $C_{L\alpha}$ is the steady flow lift-curve slope obtained from the data sheet of Ref. 7, A is the planform aspect ratio, and $\lambda_{1/2}$ is the sweep angle of the wing half-chord line in deg. The corresponding out-of-phase lift amplitude is given by Eq. (2) with $g=-D_l[1.25+0.0196\lambda_{1/2}]-D_2[0.225\,A]$, $f=6$, and D_l and D_2 are parameters which take the value $D_l=1$, $D_2=0$ for conventional wings and $D_l=0$, $D_2=1$ for delta wings.

The corresponding equations for the in-phase pitching moment about the root quarter-chord line are given by Eq. (1) with $a=0.82C_{M\alpha}$, $b=-0.085A^2\tan\lambda_{1/2}$, $d=4+A^3/32$, and $h=0.13C_{M\alpha}$. The out-of-phase pitching moment is given by Eq. (2) with $g=-0.01A\lambda_{1/2}$, and $f=6$.

$C_{M\alpha}$ denotes the steady flow pitching moment curve slopes about the wing root quarter-chord point derived from the data sheet of Ref. 7. It is to be noted that the delta wings are excluded from the pitching moment expressions.

Figures 1 and 2 illustrate the quality of the curve fitting obtained. The solid lines represent Eqs. (1) and (2) in comparison with the data plotted as symbols. These figures also indicate the variation of in- and out-of-phase lift and pitching moments with sweep angle.

The general applicability of the curve fitted expressions presented here cannot be defined precisely with confidence. A range of conventional untapered wing planforms with sweep angles between 0 and 45 deg and aspect ratios between 4 and 8 appear to be within the validity of the empirical expressions given. Lift amplitudes for delta wings with small aspect ratios of around 1-2 are also included within the expressions. Some uncertainty does exist for the validity of the expressions for aspect ratios between 2 and 4 although the trends in the lift variations suggest that the likely errors may be small. The use of the pitching moment equations for wing aspect ratio below 3 is not recommended.

It is hoped that these empirical expressions for gust loading will be of some general use for initial calculations and parametric optimization in aircraft design.

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Wake Rollup and the Kutta Condition for Airfoils Oscillating at High Frequency

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Introduction

ONE of the basic assumptions of airfoil theory deals with the presence of a stagnation point at the (usually) sharp trailing edge. This is commonly called the Kutta (or Kutta-Joukovsky) condition. While the existence of this condition is well established for steady, nonseparated flow situations, its validity for time-dependent cases is still controversial.[1-5] The Kutta condition is applied almost axiomatically in most numerical studies of unsteady wing theory,[6] but conclusions of experimentalists seem to differ, especially for higher values of the reduced frequency.

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In the present Note, we show that the Kutta condition can be applied for engineering force and moment prediction in unsteady small amplitude nonseparated flows, even when the reduced frequency is $\sigma \gg 1$. The excellent agreement between these calculations of downstream wake rollup and some smoke flow visualizations[7] indicates that the assumption of the Kutta condition is at least macroscopically correct and will suffice for integral force calculations.

Published experimental pressure data,[1-5] generally, showed good agreement between measured values and those predicted by linearized wing theory; the only deviations found were in the vicinity of the trailing edge (TE). Conclusions drawn from these data, however, differ considerably. Satyanarayana and Davis[3] concluded from their pressure measurements that the Kutta condition is not valid for reduced frequencies of $\sigma \geq 0.6$. Furthermore, Kadlec and Davis[4] argued that above this frequency, wake rollup behind oscillating airfoils is large and therefore mathematical models based on small disturbance theory are not applicable. The data of Ref. 3, however, show excellent agreement of measured and predicted pressures at 40% chord, up to their highest tested frequency ($\sigma=1.23$). Pressure measurements by Fleeter,[5] both on isolated and a cascade of flat plates, showed similar trends. On the basis of data taken at 92 and 96% of the chord, he concluded that the pressure difference at the trailing edge would be zero throughout his test conditions ($\sigma \leq 7.5$, $\alpha < 10$ deg).

Further experimental evidence in favor of the existence of the Kutta condition up to reduced frequencies of $\sigma=3.9$ were obtained[1] by placing the airfoil in the vortex street shed from a cylinder. This frequency range was further extended by Archibald[2] up to $\sigma=50$. In these papers slight deviations from linear theory pressure predictions were observed at the vicinity of the TE above $\sigma=0.6$.

Calculation of Wake Rollup

The aforementioned experimental results[1-5] show that for small TE amplitudes and very high frequencies the integral quantities such as lift are consistent with results of linearized theory. On the other hand, modified linear theory wake rollup calculations[9] are in excellent agreement when compared with flow visualization data. Furthermore, it was shown that for a given frequency, TE amplitude, and airfoil lift (circulation) history, the wake rollup is unique and the effect of this wake rollup on the lift is minimal.

In this section we present the results of a modified linear airfoil theory that shows that both lift and wake rollup of oscillating airfoils can be predicted for higher frequencies, with the aid of the Kutta condition. It is assumed that the airfoil vertical displacement $z=h(x,t)$ due to its periodic heaving and/or pitch motions is such that $h(x,t)/c \ll 1$ where c is the chord length. The TE vertical amplitude (a) is small so that no major separations occur. This limitation is a function of the Reynolds number and reduced frequency (σ) but here we assume that $a/c < 0.1$.

The wake rollup calculation is performed such that at each time interval Δt a vortex element is shed from the TE, (i.e., the circles in Fig. 1). The strength of each vortex shed γ_i is equal in magnitude and opposite in sign to the change in the circulation of the airfoil bound circulation Γ_f, during that time step. The vortex wake rollup is obtained by moving the wake elements by displacements $\Delta x_i(t)$, $\Delta z_i(t)$ such that

$$\begin{pmatrix} \Delta x_i(t) \\ \Delta z_i(t) \end{pmatrix} = \begin{pmatrix} u_i(t) \\ w_i(t) \end{pmatrix} \Delta t \qquad (1)$$